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Supersonic Turbulent Boundary Layer Subjected to Step Changes in Wall Temperature

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An investigation has been made into the effect of a step change in wall temperature on a turbulent boundary layer in a supersonic flow at a Mach number of 2.3. Measurements of the mean and turbulent flowfields were made. The results show that the relaxation behavior in general is slow except near the wall where the logarithmic behavior of the thermal and dynamical field is unaffected. Perhaps more interestingly, it appears that the effect of strong heating on the velocity field can be explained largely in terms of changes induced in the local fluid properties, and specific compressibility effects appear to be small. A mixing length argument is suggested for scaling the Reynolds stresses and the comparison with measurements of the longitudinal Reynolds stress is encouraging.

Nomenclature

C_f = skin-friction coefficient, $2\tau_w/\rho_e U_e^2$
C_h = heat transfer coefficient, $q_w/\rho_e U_e C_p(T_r - T_w)$
C_p = specific heat
h = enthalpy
l = mixing length
M, Ma = Mach number
Pr = mixed Prandtl number, $(\tau \partial h/\partial y)/(q \partial U/\partial y)$
q = heat flux
Re = Reynolds number
r = recovery factor
S = Reynolds analogy factor, $2C_h/C_f$
T = mean temperature
U = mean streamwise velocity
u', T' = fluctuating streamwise velocity and temperature
u_τ = friction velocity, $\sqrt{(\tau_w/\rho)}$
x = streamwise distance
y = wall normal distance
δ = initial boundary-layer thickness
θ_0 = scaled total temperature fluctuation [Eq. (12)]
κ = von Kármán's constant, 0.41
ρ = density
τ = shear stress

Subscripts

e = boundary-layer edge
r = recovery
s = constant stress region edge
t = stagnation condition
v = viscous layer edge
w = wall
0 = initial, undisturbed

Introduction

THE response of turbulent boundary layers to step changes in boundary conditions has been widely studied in subsonic flows (for a relatively recent review see Ref. 1). The step change may be caused by, for example, a sudden change in wall roughness, surface curvature, pressure gradient, suction/blowing, or heat transfer. The response to the change depends on the nature of the perturbation, but if the step is caused by a change in the wall boundary condition, such as a sudden change in heat transfer caused by a change in the wall temperature, the boundary layer adjusts to the new boundary condition first at the wall, and then progressively further from the wall as turbulent diffusion transmits the effects of the perturbation to the rest of the flow. These studies have great practical interest since step changes are frequently encountered in engineering practice and in nature. From a fundamental viewpoint, the initial response of the boundary layer and its subsequent relaxation to the new boundary conditions provide useful information on the time and length scales of turbulent diffusion.

For a step change in the wall boundary condition, the relaxation process is often described in terms of the growth of an internal boundary layer, which is the region near the wall where the flow scales with variables based upon the new wall condition (for example, the friction velocity and temperature based on the new level of heat transfer), whereas the rest of the boundary layer continues to scale on the variables based upon the wall conditions in the flow upstream of the step change. Typically, the internal layer grows at a rate similar to that of an undisturbed turbulent boundary layer, that is, at a rate approximately proportional to $x^{0.8}$. The relaxation rate, therefore, decreases with downstream distance, and in some instances the asymptotic state may not be reached for distances on the order of 100 initial boundary-layer thicknesses (δ_0), if at all (see, for example, the slow relaxation downstream of a prolonged region of convex curvature[2]).

To help understand the boundary-layer behavior following a sudden change in wall temperature, an experiment was performed to examine the response of a Mach 2.3 turbulent boundary layer as it entered a region where the wall was strongly heated and isothermal. In the first case considered, the ratio of wall temperature to recovery temperature, T_w/T_r, was 1.5, and in the second case T_w/T_r was 2.0. The adiabatic case ($T_w/T_r = 1.0$) was also studied to establish a reference condition. To capture the relaxation process, the response was studied over a distance of approximately 50 δ_0.

At supersonic Mach numbers, few studies of these flows have been made, but the boundary-layer response to sudden changes in heat transfer is not well known, despite the significance of such flows. Fernholz and Finley,[3,4] in their extensive review of supersonic flow data, list only a few experiments where heat transfer was important, and these cases were confined to cooled walls where the velocity and temperature boundary layers either had a common origin[5] or where upstream history effects were not defined.[6] The authors know of only one experiment with a heated, supersonic turbulent boundary layer, but the central point in that study was to

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examine the susceptibility of the layer to separation in the case of an interaction with a shock wave.[7]

For the purposes of comparison, the experiment by Kubota and Berg[8] is useful. They studied the effects of sudden changes in wall roughness (smooth to rough and rough to smooth) in a Mach 6 turbulent boundary layer. For a step change from smooth to rough, the boundary layer attained a self-preserving state in the mean flow at a distance of about 20 δ_0, and in the fluctuation profiles at about 30 δ_0. The relaxation following a step change from rough to smooth was somewhat slower with the mean flow relaxing over 28 δ_0 and the turbulent field over 40–50 δ_0. In essence, their results were not significantly different from similar work in subsonic flow.[9,10]

Now, the case of a step change in heat transfer can be somewhat different, due to the presence of two internal layers: one corresponding to the adjustment of the temperature field and the second formed by the adjustment of the velocity field (since we expect the friction velocity to be affected by the heating). These two internal layers will only be coincident when the Prandtl number is unity. Then we expect to see, at least initially, some differences appearing in the usual relationships between temperature and velocity, that is, Crocco's law for the mean field and the strong Reynolds analogy (SRA) for the fluctuations.[11,12]

One of our principal goals is to determine what role the fluid property variations play in the response of the boundary layer to a step change in wall temperature. We know that in adiabatic supersonic flows the mean flow and the turbulence intensities collapse on the subsonic correlations when the usual velocity scale based on the wall stress is modified to incorporate the effects of varying fluid temperature.[11,13] Instead of the usual velocity scale u_τ, a new velocity scale $u_\tau\sqrt{(T/T_w)}$ is used to scale the data, and the question is whether similar scaling approaches will continue to be successful when the wall is strongly heated.

Experiment

The test boundary layer was formed on the floor of the S8 wind tunnel at the Institut de Recherche sur les Phénomènes hors d'Équilibre (Fig. 1). The heated section of the wall extended over 50 cm. At the point where the wall temperature changed ($x = 0$ cm), the boundary layer was typical of a zero pressure gradient turbulent boundary layer with a thickness δ_0 of approximately 10 mm and a Reynolds number based on momentum thickness, Re_θ, of 4.1×10^3. The freestream Mach number was 2.3. The stagnation pressure was maintained at 0.5×10^5 Pa $\pm 1\%$, and the stagnation temperature was kept constant at a nominal value of 293 K. The wall temperature T_w was kept constant with time ($\pm 1\%$).

The total temperature profiles were obtained using a K-type. The recovery factor was calibrated as a function of Mach number (see Ref. 14 for details). The uncertainty in the measurement of T_0 was estimated to be less than ± 3 K.

Fluctuation measurements were made with a normal hot-wire probe using plated tungsten wire (diameter of 2.5 μm, with an active length of 0.8 mm) operating in a constant-current mode. Data reduction was based on Morkovin's modal analysis, as developed by Gaviglio[15] and others. However, since the wire Reynolds numbers were low ($2 < Re_d < 20$), the overheat ratios were low, and there were significant regions of subsonic or transonic flow near the wall, it was necessary to calibrate the probe at each overheat ratio over the entire Reynolds number and Mach number range encountered in the experiment. Further details of the calibration and data reduction procedures are given in Refs. 14, 16, and 17.

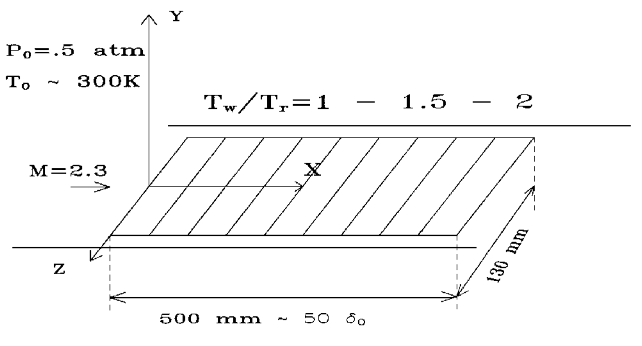

Fig. 1 Experimental configuration and coordinate system.

The anemometer overheat ratio was directly controlled by an HP 1000. The time constant was evaluated at each operating point by injecting a square wave signal and analyzing the ensemble-averaged signal. The system is described more fully by Arzoumanian and Debiève.[18] A typical frequency response of the hot wire was 200 KHz.

Analysis

Mean Flow

In a self-preserving flow (sometimes called an equilibrium flow), a relationship between velocity and temperature may be derived under the conditions that the mixed Prandtl number Pr_m is constant and $0.7 < Pr_m < 1.0$ (Refs. 19 and 20). The result is sometimes known as Crocco's law, and it is given by

$$\frac{T}{T_e} = \frac{T_w}{T_e} + \frac{T_r - T_w}{T_e}\left(\frac{U}{U_e}\right) - r\frac{(\gamma - 1)}{2}M_e^2\left(\frac{U}{U_e}\right)^2 \tag{1}$$

where r is taken to be equal to 0.89, and

$$\frac{T_r - T_w}{T_e} = 1 + r\frac{(\gamma - 1)}{2}M_e^2 - \frac{T_w}{T_e} = -\frac{q_w U_e Pr}{C_p \tau_w T_e} \tag{2}$$

Furthermore, if we consider the flow in the region where molecular transport processes can be neglected ($y \gg y_v$) and where the stress is constant, dimensional analysis[21,22] or mixing length arguments give

$$\sqrt{\frac{\rho}{\rho_w}}\frac{\partial U}{\partial y} = \frac{u_\tau}{ky} \tag{3}$$

and hence in an isobaric boundary layer,

$$I_2^+ = \int_{U_v^+}^{U^+}\sqrt{\frac{T_w}{T}}\,\mathrm{d}U^+ = \frac{1}{\kappa}\log y^+ + C_1 \tag{4}$$

where $y^+ = yu_\tau/\nu_w$, $U^+ = U/u_\tau$, and C_1 is a constant that, in principle, depends on the inner extent of the logarithmic region. It will be assumed to have a universal value of 5.25. If the temperature variation is known, either by measurement or by assuming the validity of Crocco's law, the integral in Eq. (4) may be evaluated. Furthermore, since the appropriate velocity scale for the inner and outer regions of the boundary layer is $(\rho_w/\rho)^{1/2}u_\tau$, we expect the mean (and fluctuating) velocities to scale with this variable if the flow is self-preserving. Note that when the temperature–velocity relationship is assumed as given in Eq. (1), the integral I_2^+ can be evaluated analytically (when $r = 1$ the result is the well-known van Driest transformation).

Equilibrium supersonic flows appear to follow these relationships closely, as indicated by the extensive comparisons with data given by Fernholz and Finley.[3] When the flow is not in equilibrium, as may occur downstream of a step change in wall heat transfer, the measured temperature–velocity relationship may be compared with Crocco's law to determine the degree to which the flow is out of equilibrium. By comparing the transformed velocity profile (the velocity scaled by the density-weighted friction velocity) to the well-established similarity laws for the velocity field, the degree of equilibrium achieved by the velocity field may also be determined.

Now it is also possible to derive a form of Crocco's law for perturbed flows if an equilibrium or self-preserving region can be identified. Such a region may exist within the internal layer, and one criterion for the existence of a self-preserving region could be the appearance of a logarithmic velocity variation within the internal layer. In fact, Smits and Wood[1] give examples of boundary layers perturbed by changes in pressure gradient, wall curvature, and wall roughness where logarithmic velocity profiles have been observed.

The time scale for the relaxation of the velocity field downstream of the perturbation and for the adjustment to continually changing conditions varies approximately as the turbulent kinetic energy divided by the rate of its production.[23] That is, the relaxation time varies approximately as $(\partial U/\partial y)^{-1}$. The flow near the wall, therefore, adjusts relatively quickly, and a limited region of self-preserving flow may occur. A similar argument can be made for the relaxation rate of the temperature field, bearing in mind that the Prandtl number is not unity so that the physical extents of the velocity and temperature layers are not identical. [For the velocity field, $u_\tau = \overline{q^2}/(-\overline{uv}/\partial U/\partial y)$, and by analogy for the temperature field, $T_\tau = \overline{T'^2}/(-\overline{vT}/\partial T/\partial y)$.] Within this self-preserving part of the boundary layer the velocity and temperature fields may display a logarithmic variation, and the total stress is then expected to be approximately constant over the same region.

Now, if we can identify a region of constant stress, and we assume a constant but arbitrary value of the mixed Prandtl number, it is possible to obtain for $y < y_s$, where y_s lies in the constant stress region, a Crocco's law for perturbed flows of the following form (following van Driest[19]; see Ref. 24 for details):

$$\frac{T}{T_s} = \frac{T_w}{T_s} + \frac{T_r - T_w}{T_s}\left(\frac{U}{U_s}\right) - Pr_m \frac{(\gamma - 1)}{2} M_s^2 \left(\frac{U}{U_s}\right)^2$$

we obtained

$$\frac{T}{T_w} = 1 - Pr \frac{T_\tau}{T_w} U^+ - Pr_m \frac{\gamma - 1}{2} M_\tau^2 U^{+2} \tag{5}$$

using

$$\frac{T_r - T_w}{T_s} = 1 + Pr_m \frac{(\gamma - 1)}{2} M_s^2 - \frac{T_w}{T_s} = -\frac{q_w U_s Pr}{C_p \tau_w T_s} \tag{6}$$

where $M_s = U_s/\sqrt{(\gamma R T_s)}$, $M_\tau^2 = u_\tau^2/(\gamma R T_w)$, and $U_s = U(y_s)$. Equation (5) can be applied to perturbed flows that display a constant stress region. Furthermore, in the region where molecular transport processes can be neglected ($y \gg y_v$) and where the stress is constant, Eqs. (3) and (4) still apply as long as the scaling is based on the local wall values. Thus, Eqs. (4) and (5) completely define the mean velocity and temperature fields for $y_v \ll y < y_s$.

If we assume that the presence of a logarithmic velocity profile indicates the presence of a constant stress region (this does not necessarily follow, but for the present purpose it may be a reasonable approximation), then Eq. (5) may be used to determine the heat transfer at the wall by fitting the curve to the data in the region $y_v \ll y < y_s$. This method for finding the friction temperature $T_\tau (\equiv q_w/\rho_w C_p u_\tau)$ is similar to the Clauser method for finding the friction velocity u_τ from the measured velocity profile (see Ref. 25 for a discussion of the Clauser chart method in supersonic boundary layers with and without strong perturbations).

The procedure for obtaining the skin-friction and heat transfer coefficients from the experimental profiles follows an iterative solution of Eqs. (4) and (5). First, an estimate for u_τ is found by fitting the experimental velocity and temperature data to Eq. (4) in the logarithmic region. The extent of the logarithmic region may not be known a priori, and choosing the experimental points for the curvefit becomes part of the iteration procedure. Second, this estimate for u_τ is used in fitting Eq. (5) to the measured temperature and velocity data for $y < y_s$, thereby deriving an estimate for T_τ. Third, the estimate for T_τ is used to curvefit the experimental profile of T/T_w, whereby enabling a better estimate for u_τ to be found using Eq. (5). In practice we found that the first estimates for u_τ and T_τ were always within 1% of the final values, and an iteration was unnecessary. It should, in principle, also be possible to find Pr_m using this iterative procedure. In practice, the estimates of T_τ and C_h are rather insensitive to the value of Pr_m, which makes it difficult to find Pr_m accurately from the data. In accordance with previous experience (see, for example, Ref. 3), it was then assumed that $Pr_m = 0.86$ for all data reduction. In contrast, the constant C_3 need not be known a priori and may be found from the data with reasonable accuracy.

To express the temperature profile in the more familiar logarithmic form, we note that Eq. (5) may be rewritten using a turbulent total temperature T_i similar to that suggested by Michel et al.[26] and defined by $T_i = T + (Pr_m/2C_p)U^2$. This leads to

$$\frac{T_i}{T_w} = 1 - \frac{Pr_m T_\tau}{T_w} U^+ \tag{7}$$

Hence

$$\frac{\partial T_i}{\partial y} = -Pr_m T_\tau \frac{\partial U^+}{\partial y} = -\frac{Pr_m T_\tau}{\kappa y}\sqrt{\frac{T}{T_w}} \tag{8}$$

and

$$I_3^+ = \int_0^{T_i^+} \sqrt{\frac{T_w}{T}}\, \mathrm{d}T_i^+ = \frac{1}{k}\log y^+ + C_3 \tag{9}$$

where $T_i^+ = -T_i/(T_\tau Pr_m)$ and C_3 again depends on the inner limits of the logarithmic region. It will also depend on the Prandtl number and the ratio of the wall temperature to the recovery temperature T_w/T_r (Ref. 24). Michel et al.[26] found that $C_3 = 3.6$ for their flow conditions.

Turbulence Scaling

The same arguments used to establish Eq. (4) can also be used to derive a scaling for the velocity profile based upon outer flow parameters, e.g., δ. In this case, the scaling will apply to the region of the boundary layer between the viscous region and the boundary layer edge. An integral expression analogous to Eq. (4) can be written as[3]

$$\int_{U_v^+}^{U_e^+} \sqrt{\frac{T_w}{T}}\, \mathrm{d}U^+ = g(\eta) \tag{10}$$

where $\eta \equiv y/\delta$.

Now consider the turbulence shear stress τ in a compressible, turbulent boundary layer expressed as $\tau = -\overline{\rho uv}$. Using the classical mixing length arguments, we can write $\tau/\tau_w = (T_w/T)(l^2/u_\tau^2)(\partial U/\partial y)^2$. Or after introducing η and using Eq. (10),

$$\frac{\tau}{\tau_w} = \left(\frac{l}{\delta}\frac{\partial g}{\partial \eta}\right)^2 \tag{11}$$

This analysis will apply to the flow in the outer region of the boundary layer, and if l/δ and $\partial g/\partial \eta$ are universal functions of η, then it follows that τ/τ_w is also a universal function of η. Furthermore, τ_w then becomes the proper scaling for τ, and also for the turbulence intensities if the anisotropy of the flow is similar to its subsonic counterpart. This consideration leads to the well-known representation developed by Morkovin[11] for examining the influence of the Mach number on the Reynolds stresses: $(\overline{\rho u^2}/\tau_w) = f(y/\delta)$. There, the variation in density in Eq. (10) was from the Mach number gradient in the boundary layer. Here, the density variation is also affected by the heating at the wall.

Results

Mean Flow

The effect of heating on the boundary-layer integral scales is an increase of 33% in the displacement thickness δ^* and a decrease of 10% in the momentum thickness θ, which results in a 40% increase in the compressible form factor. The value of δ used in the following figures was chosen as the point where the total pressure reached 98% of the freestream value.

Two total temperature profiles in the heated boundary layer are given in Fig. 2. The profiles were measured at positions of 8 and 46 cm downstream of the beginning of the heated wall. The internal thermal layer grows relatively rapidly such that at $x = 8$ cm it fills about half the boundary layer, and by $x = 46$ cm the temperature and the velocity layers are almost coincident. A comparison with Crocco's law [Eq. (1)] indicates that the boundary layer at $x = 8$ cm is strongly perturbed by the heating and by $x = 46$ cm has not yet reached equilibrium (Fig. 3). For a lower heating ($T_w/T_r = 1.5$), equilibrium with temperature is yet reached in this last section. In contrast, far from the wall, the velocity profiles indicate that the temperature field has only a mild effect on the velocity distribution.

The iterative solution described in detail earlier was used to find the variations of wall friction and heat transfer along the plate. This process requires fitting the experimental data to analytical relationships. Figures 4 and 5 show the fits of velocity and temperature data with the logarithmic law of Eq. (4) and Crocco's law, respectively. The coefficients C_f and C_h were then obtained from the iterative solution of Eqs. (4) and (5). The results are plotted in

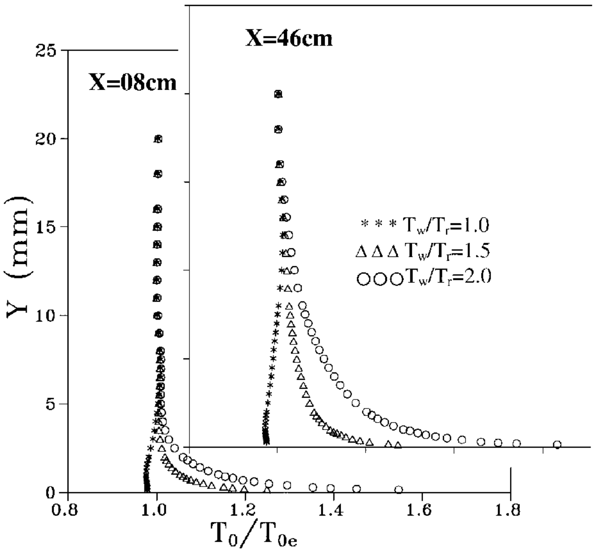

Fig. 2 Total temperature profiles at $x = 8$ and 46 cm.

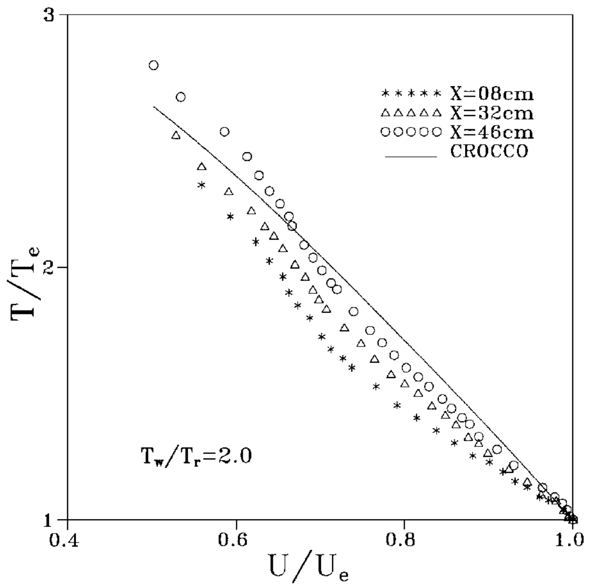

Fig. 3 Comparison of measured temperature and velocity data with Crocco's law [Eq. (1)] at $x = 8$, 32, and 46 cm.

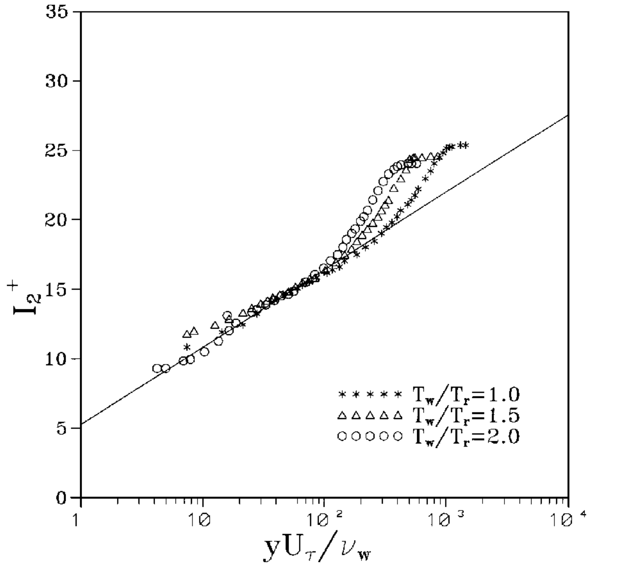

Fig. 4 Semilogarithmic representation of transformed velocity profiles at $x = 32$ cm with u_T adjusted to give the best fit to the log-law [Eq. (4)].

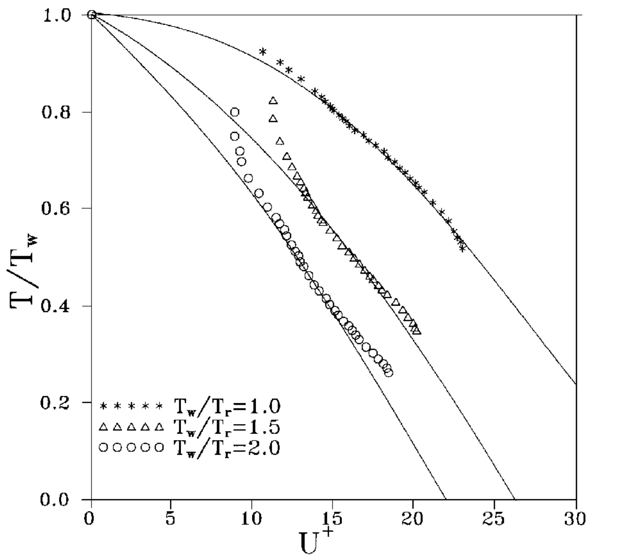

Fig. 5 Temperature–velocity profiles at $x = 32$ cm with T_T adjusted to give the best fit to Eq. (5).

Figs. 6 and 7. The effect of heating is clearly seen to reduce the skin-friction and heat transfer coefficients, regardless of the level of relaxation. However, heating will change the fluid properties near the wall. Then, if we assume that the skin-friction coefficient follows the same relation as that found for adiabatic flows, provided that the density and viscosity are evaluated using near-wall conditions rather than external conditions,[27] $C_f \propto R_{\theta i}^{-1/5} g(T_*/T_e)$, where $R_{\theta i}$ is the Reynolds number based on the momentum thickness θ_i, $g = [(\mu_*/\mu_e)^{1/5}/(T_*/T_e)]$, where the asterisk corresponds to a reference temperature[28] and $\mu(T)$ is the viscosity, we find that most of the differences in C_f observed due to heating are due simply to the change in fluid properties.

The argument is similar to that advanced by Monaghan[28] and Hinze[29] to explain the decrease in C_f with Mach number for adiabatic flows. In that case also, the increase in Mach number increases the ratio between the wall and external temperatures, and the resulting change in the fluid properties is sufficient to explain the observed decrease in C_f. Application of this formula in the heated case, although it underestimates C_f, gives the most part of the observed decrease in C_f. The difference with the observed value can be attributed to the fact that the formula can be applied only to equilibrium turbulent boundary layer and without the least pressure gradient. A similar argument can also be made for the heat transfer coefficient. Finally, it appears that the Reynolds analogy factor S is out of equilibrium: it decreases continuously and reaches a value of about 1.5 near the end of the test section, when a classical value is 1.24. The skin-friction coefficients were also determined using the momentum integral equation and the values were within 10% of those shown in Fig. 6.

The velocity and temperature profiles near the wall are given in Figs. 4 and 8 in semilogarithmic form. When the velocity profile near the wall is scaled on density, the variation in density due to the combined effect of compressibility and heating simply alters the velocity scale for the velocity profile without introducing any explicit effects due to the strong heating. That is, there is no effect of buoyancy since in a supersonic flow it is the kinetic energy that is important. Furthermore, the relaxation of the temperature profile and the large extent of the logarithmic region are clearly evident in Fig. 8. For this flow, we found the constant C_3 to be approximately equal to 3.0, which is a little lower than the value found by Michel et al.[26] Finally, the velocity distribution in the wake region is presented in Fig. 9. The success of the scaling used in this region supports the turbulence scaling discussed earlier.

Turbulence

Previous laser Doppler anemometry (LDA) measurements of the longitudinal velocity fluctuations in the same flow[30] are shown in

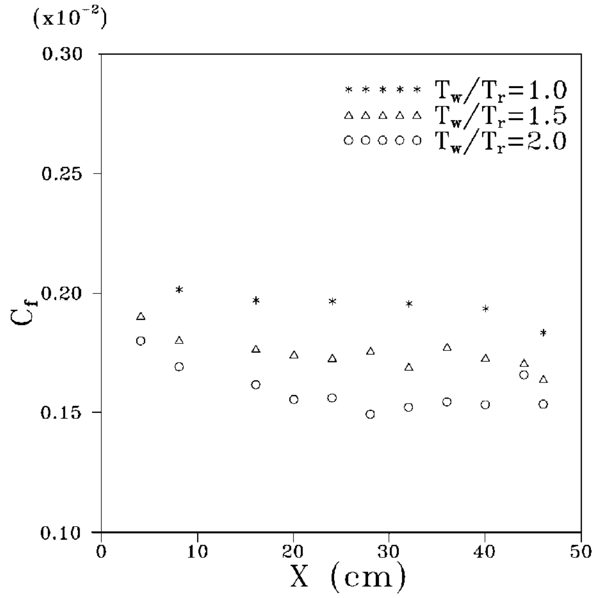

Fig. 6 Distribution of skin-friction coefficients for three different wall temperatures.

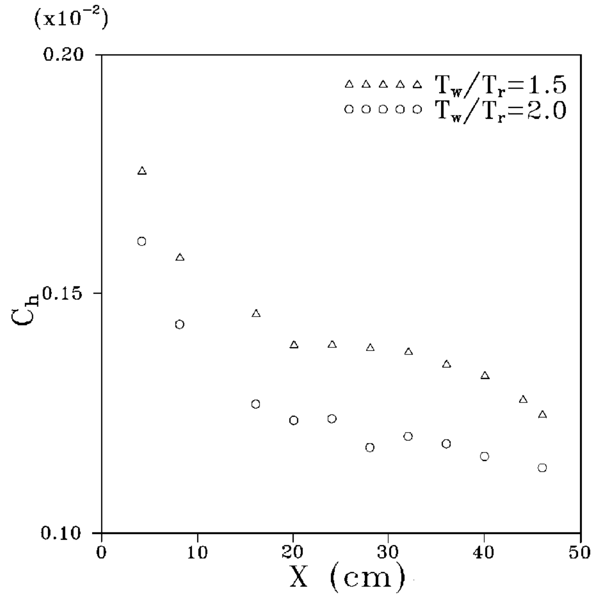

Fig. 7 Distribution of heat transfer coefficients for three different wall temperatures.

Fig. 10. The dashed line represents the measurements by Klebanoff[31] in an unheated, incompressible turbulent boundary layer. In this figure, the velocity fluctuations are scaled using Morkovin's representation. This scaling has been shown experimentally to be generally applicable over a wide range of Mach numbers ($0 < Ma < 5$) and also, as expected from the previous analysis, appears to take into account the effects of wall heating. That is to say that the effects of heating are similar to the effects of Mach number: the data can be collapsed using a scaling based solely on fluid property variations. The disagreement between the two measured profiles is likely due to the difficulty associated with accurately determining the boundary-layer thickness in this perturbed flow. That is, the boundary-layer edge becomes obscured as gradients in the boundary layer become indistinguishable from gradients in the external flow near the boundary-layer edge.

The influence of the wall heating on the fluctuations in the total temperature are shown in Fig. 11. The increase in the turbulence intensity of the total temperature fluctuations with heating is large, and the growth of the thermal boundary layer is clearly evident. It is not obvious at first how these profiles scale. One possibility is the temperature difference $(T_w - T_{0e})$, but in an adiabatic flow this

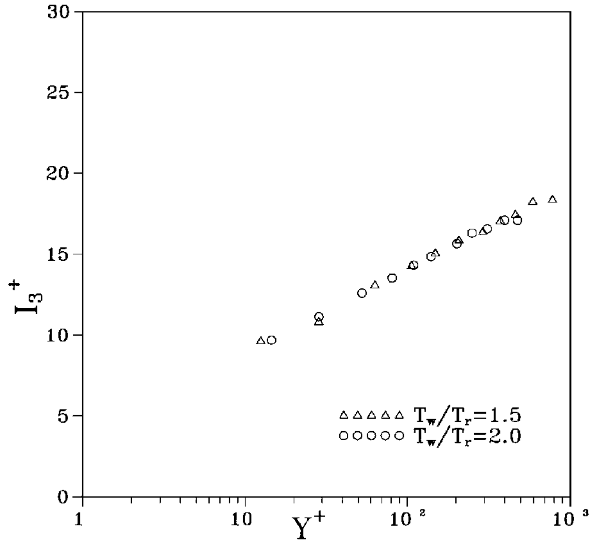

Fig. 8 Semilogarithmic representation of temperature profiles at x = 32 cm for = 1.5 and 2.0.

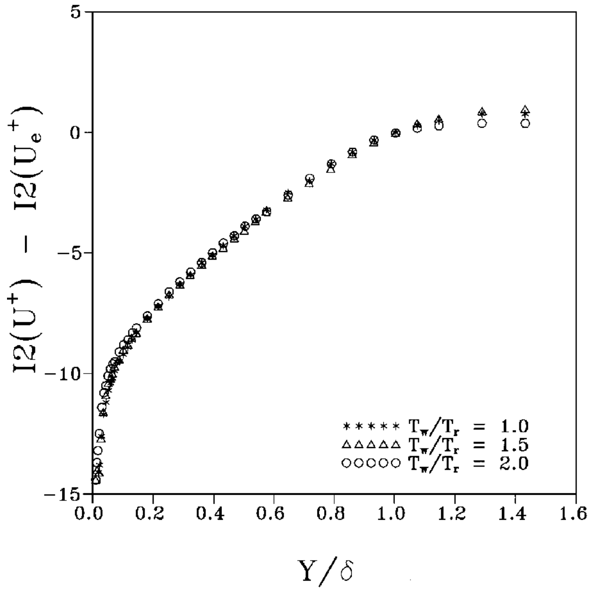

Fig. 9 Deficit law representation of the velocity profiles at x = 46 cm.

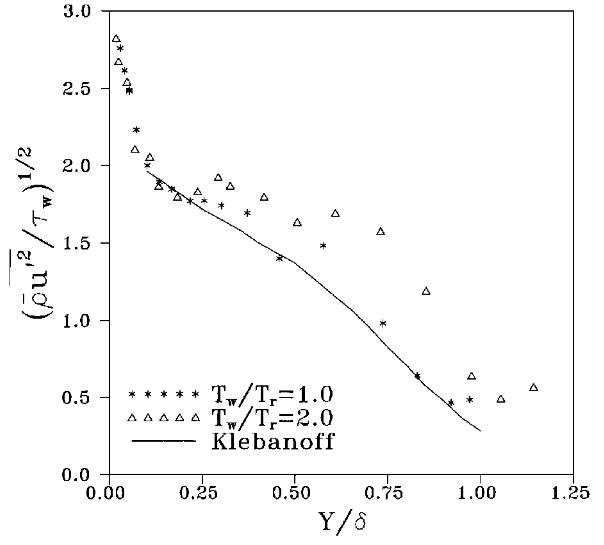

Fig. 10 Longitudinal velocity fluctuations at x = 32 cm (Ref. 24).

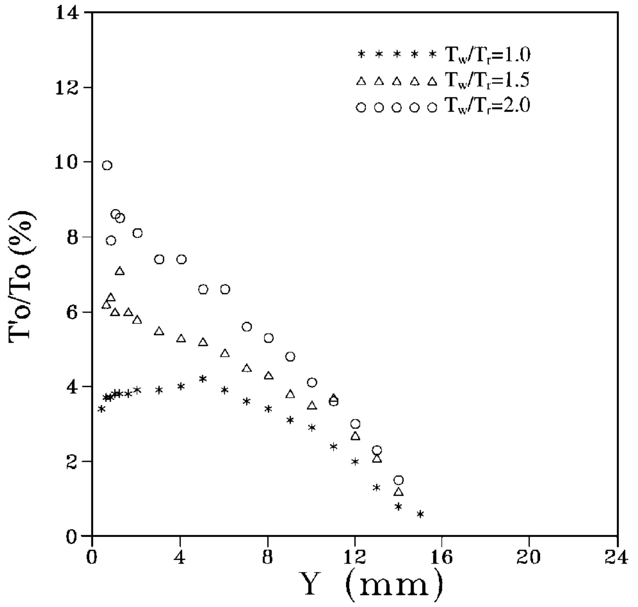

Fig. 11 Total temperature fluctuations at x = 32 cm.

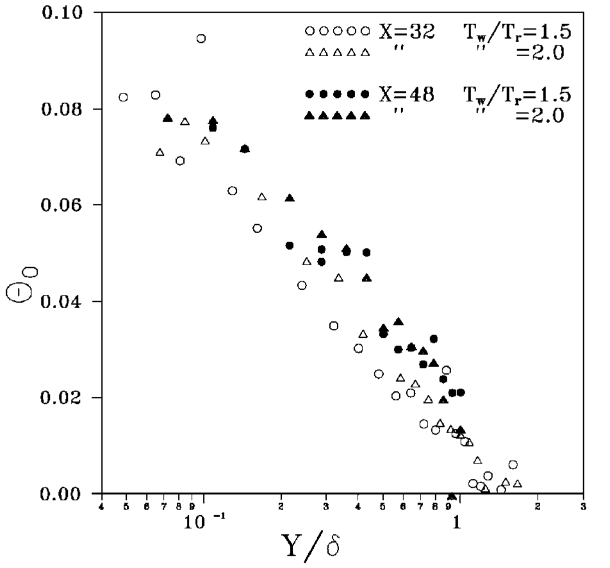

Fig. 12 Excess total temperature fluctuations at x = 32 and 48 cm scaled by the temperature difference.

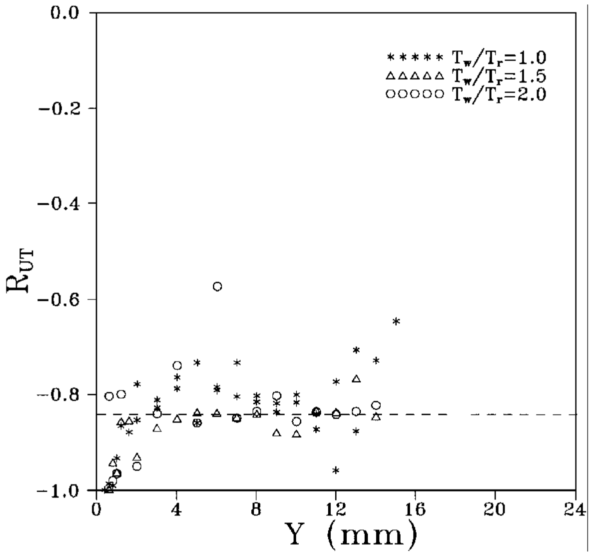

Fig. 13 Velocity–temperature correlation at x = 32 and 48 cm.

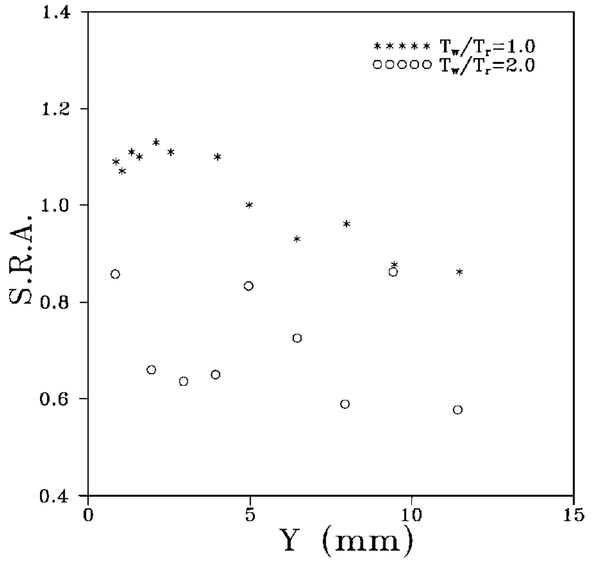

Fig. 14 Profiles of the strong Reynolds analogy at x = 32 cm.

temperature difference is small and total temperature fluctuations are still significant, and therefore this scaling does not seem appropriate for adiabatic flows. However, with a heated wall this scaling may be useful, as long as we consider the increase in the fluctuation level in the heated flow (h) relative to the adiabatic case (aw). Hence, by analogy with the turbulent velocity scaling, we can define a new nondimensional rms total temperature fluctuation level:

$$\theta_0 = \frac{\sqrt{\overline{T_0'^2}}\Big|_h - \sqrt{\overline{T_0'^2}}\Big|_{aw}}{T_w - T_{0e}} \tag{12}$$

This scaling appears to give a reasonable collapse of the experimental data, as may be seen in Fig. 12.

For the turbulent heat transfer, we are interested in the behavior of the transverse heat flux $\overline{(\rho v)'T'}$. Unfortunately, this is a difficult quantity to measure directly, particularly under our experimental conditions. Some indication can be given, however, by examining the behavior of the longitudinal heat flux $\overline{(\rho u)'T'}$. From Fig. 13, it appears that the correlation coefficient $-R_{uT}$ is not significantly affected by the heating. The correlation remains almost unchanged from its adiabatic value in a supersonic flow of 0.8–0.9. This value may be compared with a typical value of 0.4–0.5 in a heated subsonic flow.

This high degree of correlation between the velocity and temperature (even though the flow is heated) suggests that the instantaneous scales of u and T are connected by a relationship such as $T = f(u)$. In an adiabatic flow, for example, an instantaneous law of this form is supplied by the thermodynamic hypotheses $T_0' = 0$, which lead to the expression of the SRA:

$$(\gamma - 1)Ma^2 \frac{\sqrt{\overline{u'^2}}/\bar{U}}{\sqrt{\overline{T'^2}}/\bar{T}} = 1$$

and

$$R_{uT} = -1$$

In heated flows where $\overline{T_0'^2}$ is not negligible (Fig. 11), this hypothesis cannot be used.

When the distributions $\bar{T}(y)$ and $\bar{U}(y)$ are known, a mean relation $\bar{T} = g(\bar{U})$ may be established. A linearization for small fluctuations can then be obtained with the aid of a mixing-length style gradient scheme, $u' = l_u \partial\bar{U}/\partial y$ and $T' = l_t \partial\bar{T}/\partial y$, or in a direct linearization of the mean law $\bar{T} = g(\bar{U})$. In the case of $l_U(y/\delta) = l_T(y/\delta)$, that is, when the Prandtl number defined in terms of u' and T' is equal to unity, the two methods lead to the same result: the SRA.[32] For a direct linearization of the mean relationship $\bar{T} = g(\bar{U})$, similar mixing length arguments are necessary. This second method can be found in Cebeci and Smith,[33] who linearized Crocco's simple law (recovery factor is unity). To evaluate the influence of wall heating on the SRA, we can linearize Crocco's modified law (recovery factor different from unity), which explicitly includes the heating parameter T_w/T_r:

$$\frac{(\gamma-1)Ma^2u\prime/U}{T\prime/T} = -\left\{r + \left(\frac{T_w}{T_r} - 1\right)\right.$$
$$\left.\times\left[\frac{1 + r\,Ma_e^2(\gamma-1)/2}{Ma_e^2(\gamma-1)}\right]\left(\frac{1}{U/U_e}\right)\right\}^{-1} \tag{13}$$

Here, the second term on the right-hand side introduces the influence of heating. This term also depends on y/δ through the velocity profile, U/U_e, which has a slight dependence on the heating. In adiabatic flows, the right-hand side reduces to the recovery factor, $r = 0.89$. But in the case where $T_w/T_r := 2$, we can expect that, for a point located in the middle of the layer, the right-hand side of Eq. (13) will be reduced by a factor of about 2 with respect to the adiabatic value. The hot-wire data show this decrease from the adiabatic value (Fig. 14). However, the overestimate of the velocity fluctuations by a hot wire in the heated case leads to an overestimate of this ratio, and the data are not accurate enough to verify the validity of Eq. (13).

Conclusions

The mean and fluctuating velocity and temperature fields were measured in a supersonic turbulent boundary layer experiencing a step change in wall temperature. To describe the adaptation of the boundary-layer flow to the new conditions at the wall, a scaling based on mixing length arguments was proposed.

For the outer region of the boundary layer, a deviation from Crocco's law was observed, and the recovery of this relation was not yet complete at a position 50 δ_0 downstream of the start of heating. However, the temperature field appeared to have had little effect on the velocity field in this same region of the boundary layer. Near the wall, that is, in the internal layer, logarithmic variations in the mean velocity and temperature profiles were found when the scaling arguments took into account the new values of density and friction velocity. Similarly, the change in fluid properties at the heated wall can explain the observed decrease in the skin-friction and wall heat transfer coefficients.

A scaling of the Reynolds stresses was suggested following the same arguments proposed for the mean flowfield, that is, a mixing length hypothesis. This scaling was first developed to account for the effects of Mach number in a supersonic turbulent boundary layer on an adiabatic wall. With this scaling, the streamwise Reynolds stress profile for the boundary layer on the heated wall collapsed onto the upstream profile. A gradient hypothesis was used to scale the total temperature fluctuations. This also resulted in a collapse of the profiles. Finally, the correlation between velocity and temperature fluctuations appeared unaffected by the heating at the wall.

In general, the observed changes in the boundary layer due to the heating can be explained in terms of variations in fluid properties and the new boundary conditions at the wall.

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